Examination – FK5024

Nuclear & particle physics, astrophysics & cosmology

August 28, 2019, 08:00 – 13:00

DM, JG

1 (4p) Consider the following decays/reactions (the particles are not bound or virtual). Discuss which of these it is possible to observe and draw a Feynman diagram for that case. If a process is impossible, state a conservation law which is violated.

(a) $W^+ \to e^+ + \nu_e$

(b) $n \to \pi^+ + \pi^- + \pi^0$

(c) $\tau^+ \to \mu^+ + \nu_\mu + \bar{\nu}_\tau$

(d) $\mu^+ + \mu^- \to \pi^+ + \pi^- + \pi^0$

2 (4p)

In a scattering experiment, an electron with energy 27.5 GeV collides head-on with a proton of energy 820 GeV. The electron is scattered at an angle of 150^o with respect to the direction of the incoming proton. The energy of the electron after the collision is 21.5 GeV.

(a) The four-vector Q is defined as being the four-momentum of the exchange photon. Calculate Q^2.

(b) It is assumed that a single quark in the proton takes part in the interaction. If that quark carried 40% of the proton's momentum before taking part in the interaction, calculate the momentum of the quark after the interaction.

(3) (4p)
The semi-empirical mass formula (SEMF) can be used to calculate nuclear masses to a good precision over a wide range of mass numbers A.
The SEMF gives the binding energy, $B(A,Z)$, for a nucleus with atomic number Z and mass number A as :

$$B(A,Z) = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_a \frac{(Z-\frac{A}{2})^2}{A} + a_p A^{-\frac{1}{2}} \quad ,$$

Explain the physical origin of each term in the SEMF.

(4) (4p) (a) A sample of a radioactive material decays via α-emission. If the α-particles have an energy of 4 MeV, estimate the half-life of the nuclei.

(b) Explain qualitatively the origin of α radioactive decays.

(5) (4p)
Describe the early work that Hubble, Leavitt and others did to establish what is now called the Hubble law. What does the Hubble law describe?
What value didHubble determine for the Hubble constant ?
What do modern Hubble constant measurements suggest?

A de Sitter universe is a universe that is dominated by vacuum energy, a positive cosmological constant. Such a universe is characterized by:

$$\rho = \frac{\Lambda}{8\pi G} \equiv \rho_\Lambda \quad , \quad p = -\rho_\Lambda$$

Assuming that this universe is flat, $k = 0$, show that the scale factor , $a(t)$, increases exponentially in such a universe.

(6) (4p)

A matter dominated universe can be descried by $a(t) = t^{2/3}$. Assuming that $H_0 = 70$ km/s/Mpc, use this relation to estimate the current age of the universe.
Name four types of observations that indicate the presence of dark matter in the universe.
How can we explain the presence of a dipole in the temperature of the comic microwave background ?

Useful equations

Friedmann equation (here $c = 1$):

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}(\rho_m+\rho_r+\rho_\Lambda) - \frac{k}{a^2} = H_0^2\Big[\Omega_M(1+z)^3+\Omega_R(1+z)^4+\Omega_K(1+z)^2+\Omega_\Lambda\Big],$$

with $\rho_\Lambda = \frac{\Lambda}{8\pi G_N}$, $\Omega_M = \rho_m(t_0)/\rho_c^0$, etc, also $\rho_k = -\frac{3k}{8\pi G_N a^2}$, $\Omega_K = \rho_k(t_0)/\rho_c^0$.

The critical density at the present time $t = t_0$ is

$$\rho_c^0 = (3H_0^2)/(8\pi G_N) = 1.05h^2 \cdot 10^{-5} \text{ GeV cm}^{-3}.$$

Fluid equation:
$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$

Equation of state:
$p/c^2 = w \cdot \rho$
Acceleration equation:
$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\left(\rho + 3\frac{p}{c^2}\right)$

For a sum of different components ρ_i in the universe, $p_i/c^2 = w_i\rho_i$
$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\sum_i\left(\rho_i + 3\frac{p_i}{c^2}\right) = -\frac{4\pi G_N}{3}\sum_i \rho_i(1+3w_i)$

"Deceleration parameter":

$$q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)}\frac{1}{H_0^2}$$

Coulomb potential:

$$V(r) = \frac{Q_1Q_2}{4\pi\epsilon_0 r} = \frac{Z_1Z_2\alpha\hbar c}{r},$$

where $Q_i = Z_i e$, $\alpha = \frac{1}{137}$ is the fine-structure constant and

$$\hbar c = 1.973 \cdot 10^{-7} \text{ eV·m}$$

Geiger-Nuttall relation:

$$\log_{10}\lambda = C + DE_\alpha^{-\frac{1}{2}}$$

$$C \sim 52, \quad D \sim 140 \text{ (MeV)}^{\frac{1}{2}}$$

Constants (W, Z masses and Hubble h are 2017 values)

Name	Symbol	value
Newton's constant	G_N	$6.672 \cdot 10^{-11}$ $\mathrm{m^3 kg^{-1} s^{-2}}$
Speed of light	c	$2.998 \cdot 10^{8}$ m $\mathrm{s^{-1}}$ or $3.076 \cdot 10^{-7}$ Mpc $\mathrm{year^{-1}}$
Planck's constant	$\hbar = h/2\pi$	$1.055 \cdot 10^{-34}$ $\mathrm{m^2}$ kg $\mathrm{s^{-1}}$
Boltzmann's constant	k_B	$1.381 \cdot 10^{-23}$ J/K or $8.619 \cdot 10^{-5}$ eV/K
Radiation constant	$\alpha_{rad} = \pi^2 k_B^4 / 15\hbar^3 c^3$	$7.565 \cdot 10^{-16}$ J $\mathrm{m^{-3}}$ $\mathrm{K^{-4}}$
Electron rest mass energy	$m_e c^2$	0.511 MeV
Proton rest mass energy	$m_p c^2$	938.3 MeV
Neutron rest mass energy	$m_n c^2$	939.6 MeV
W boson rest mass energy	$m_W c^2$	80.4 GeV
Z boson rest mass energy	$m_Z c^2$	91.2 GeV
Planck energy	$M_{Pl} c^2$	$1.2 \cdot 10^{19}$ GeV
Thomson cross section	σ_e	$6.652 \cdot 10^{-29}$ $\mathrm{m^2}$
Neutron half-life (free neutron)	$t_{\frac{1}{2}}$	611 s
Hubble constant	H_0	$100 \cdot h$ km $\mathrm{s^{-1}}$ $\mathrm{Mpc^{-1}}$
	h	0.70 ± 0.03
Inverse Hubble constant	H_0^{-1}	$9.77 h^{-1} \cdot 10^9$ years
Critical density	ρ_c^0	$1.05 h^2 \cdot 10^{-5}$ GeV $\mathrm{cm^{-3}}$

Conversion factors

1 pc = 3.261 light-years = $3.086 \cdot 10^{16}$ m
1 AU = $1.5 \cdot 10^{11}$ m
1 year = $3.156 \cdot 10^{7}$ s
1 eV = $1.602 \cdot 10^{-19}$ J
1 $\mathrm{M_\odot}$ = $1.989 \cdot 10^{30}$ kg

PARTICLE DATA

(Mass in MeV/c²; Lifetime in Seconds; Charge in Units of Proton Charge.)

QUARKS (Spin $\frac{1}{2}$)

	Flavor	Charge	Mass (speculative)		
			Bare	Effective	
				In baryons	In mesons
First generation	d	$-\frac{1}{3}$	7.5	363	310
	u	$+\frac{2}{3}$	4.2		
Second generation	s	$-\frac{1}{3}$	150	538	483
	c	$+\frac{2}{3}$	1100	1500	
Third generation	b	$-\frac{1}{3}$	4200	4700	
	t	$+\frac{2}{3}$	>23,000		

LEPTONS (Spin $\frac{1}{2}$)

	Lepton	Charge	Mass	Lifetime	Principal decays
First generation	e	−1	0.511003	∞	—
	ν_e	0	~ 0	∞	—
Second generation	μ	−1	105.659	2.197×10^{-6}	$e\nu_e\bar{\nu}_e$
	ν_μ	0	~ 0	∞	—
Third generation	τ	−1	1784	3.3×10^{-13}	$\mu\nu_\mu\bar{\nu}_\mu$, $e\nu_e\bar{\nu}_e$, $\rho\nu_\tau$
	ν_τ	0	~ 0	∞	—

MEDIATORS (Spin 1)

Mediator	Charge	Mass	Lifetime	Force
gluon	0	0	∞	strong
photon (γ)	0	0	∞	electromagnetic
$W^\pm$	±1	81,800	unknown	(charged) weak } electroweak
Z^0	0	92,600	unknown	(neutral) weak

BARYONS (Spin $\frac{1}{2}$)

Baryon	Quark content	Charge	Mass	Lifetime	Principal decays
N $\begin{cases} p \\ n \end{cases}$	uud	+1	938.280	∞	—
	udd	0	939.573	900	$pe\bar{\nu}_e$
Λ	uds	0	1115.6	2.63×10^{-10}	$p\pi^-$, $n\pi^0$
Σ^+	uus	+1	1189.4	0.80×10^{-10}	$p\pi^0$, $n\pi^+$
Σ^0	uds	0	1192.5	6×10^{-20}	$\Lambda\gamma$
Σ^-	dds	−1	1197.3	1.48×10^{-10}	$n\pi^-$
Ξ^0	uss	0	1314.9	2.90×10^{-10}	$\Lambda\pi^0$
Ξ^-	dss	−1	1321.3	1.64×10^{-10}	$\Lambda\pi^-$
Λ_c^+	udc	+1	2281	2×10^{-13}	not established

BARYONS (Spin $\frac{3}{2}$)

Baryon	Quark content	Charge	Mass	Lifetime	Principal decays
Δ	uuu, uud, udd, ddd	+2, +1, 0, −1	1232	0.6×10^{-23}	$N\pi$
Σ^*	uus, uds, dds	+1, 0, −1	1385	2×10^{-23}	$\Lambda\pi$, $\Sigma\pi$
Ξ^*	uss, dss	0, −1	1533	7×10^{-23}	$\Xi\pi$
Ω^-	sss	−1	1672	0.82×10^{-10}	ΛK^-, $\Xi^0\pi^-$, $\Xi^-\pi^0$

PSEUDOSCALAR MESONS (Spin 0)

Meson	Quark content	Charge	Mass	Lifetime	Principal decays
$\pi^\pm$	$u\bar{d}$, $d\bar{u}$	+1, −1	139.569	2.60×10^{-8}	$\mu\nu_\mu$
π^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	0	134.964	8.7×10^{-17}	$\gamma\gamma$
$K^\pm$	$u\bar{s}$, $s\bar{u}$	+1, −1	493.67	1.24×10^{-8}	$\mu\nu_\mu$, $\pi^\pm\pi^0$, $\pi^\pm\pi^-\pi^+$
K^0, $\bar{K}^0$	$d\bar{s}$, $s\bar{d}$	0, 0	497.72	K_S^0 0.892×10^{-10}; K_L^0 5.18×10^{-8}	$\pi^+\pi^-$, $\pi^0\pi^0$; $\pi e\nu_e$, $\pi\mu\nu_\mu$, $\pi\pi\pi$
η	$(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$	0	548.8	7×10^{-19}	$\gamma\gamma$, $\pi^0\pi^0\pi^0$, $\pi^+\pi^-\pi^0$
η'	$(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$	0	957.6	3×10^{-21}	$\eta\pi\pi$, $\rho^0\gamma$
$D^\pm$	$c\bar{d}$, $d\bar{c}$	+1, −1	1869	9×10^{-13}	$K\pi\pi$
D^0, $\bar{D}^0$	$c\bar{u}$, $u\bar{c}$	0, 0	1865	4×10^{-13}	$K\pi\pi$
$F^\pm$ (now $D_s^\pm$)	$c\bar{s}$, $s\bar{c}$	+1, −1	1971	3×10^{-13}	not established
$B^\pm$	$u\bar{b}$, $b\bar{u}$	+1, −1	5271	14×10^{-13}	$D + ?$
B^0, $\bar{B}^0$	$d\bar{b}$, $b\bar{d}$	0, 0	5275		
η_c	$c\bar{c}$	0	2981	6×10^{-23}	$KK\pi$, $\eta\pi\pi$, $\eta'\pi\pi$

VECTOR MESONS (Spin 1)

Meson	Quark content	Charge	Mass	Lifetime	Principal decays
ρ	$u\bar{d}$, $d\bar{u}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$	+1, −1, 0	770	0.4×10^{-23}	$\pi\pi$
K^*	$u\bar{s}$, $s\bar{u}$, $d\bar{s}$, $s\bar{d}$	+1, −1, 0, 0	892	1×10^{-23}	$K\pi$
ω	$(u\bar{u} + d\bar{d})/\sqrt{2}$	0	783	7×10^{-23}	$\pi^+\pi^-\pi^0$, $\pi^0\gamma$
ϕ	$s\bar{s}$	0	1020	20×10^{-23}	K^+K^-, $K^0\bar{K}^0$
J/ψ	$c\bar{c}$	0	3097	1×10^{-20}	e^+e^-, $\mu^+\mu^-$, 5π, 7π
D^*	$c\bar{d}$, $d\bar{c}$, $c\bar{u}$, $u\bar{c}$	+1, −1, 0, 0	2010	$>1 \times 10^{-22}$	$D\pi$, $D\gamma$
Υ	$b\bar{b}$	0	9460	2×10^{-20}	$\tau^+\tau^-$, $\mu^+\mu^-$, e^+e^-