

Useful equations

Friedmann equation (here $c = 1$):

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}(\rho_m + \rho_r + \rho_\Lambda) - \frac{k}{a^2} = H_0^2 \left[\Omega_M(1+z)^3 + \Omega_R(1+z)^4 + \Omega_K(1+z)^2 + \Omega_\Lambda \right],$$

with $\rho_\Lambda = \frac{\Lambda}{8\pi G_N}$, $\Omega_M = \rho_m(t_0)/\rho_c^0$, etc, also $\rho_k = -\frac{3k}{8\pi G_N a^2}$, $\Omega_K = \rho_k(t_0)/\rho_c^0$.

The critical density at the present time $t = t_0$ is

$$\rho_c^0 = (3H_0^2)/(8\pi G_N) = 1.05h^2 \cdot 10^{-5} \text{ GeV cm}^{-3}.$$

Fluid equation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

Equation of state:

$$p/c^2 = w \cdot \rho$$

Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\left(\rho + 3\frac{p}{c^2}\right)$$

For a sum of different components ρ_i in the universe, $p_i/c^2 = w_i\rho_i$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\sum_i\left(\rho_i + 3\frac{p_i}{c^2}\right) = -\frac{4\pi G_N}{3}\sum_i\rho_i(1 + 3w_i)$$

”Deceleration parameter”:

$$q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)}\frac{1}{H_0^2}$$

Coulomb potential:

$$V(r) = \frac{Q_1 Q_2}{4\pi\epsilon_0 r} = \frac{Z_1 Z_2 \alpha \hbar c}{r},$$

where $Q_i = Z_i e$, $\alpha = \frac{1}{137}$ is the fine-structure constant and

$$\hbar c = 1.973 \cdot 10^{-7} \text{ eV}\cdot\text{m}$$

Constants (W, Z masses and Hubble h are 2017 values)

Name	Symbol	value
Newton's constant	G_N	$6.672 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Speed of light	c	$2.998 \cdot 10^8 \text{ m s}^{-1}$ or $3.076 \cdot 10^{-7} \text{ Mpc year}^{-1}$
Planck's constant	$\hbar = h/2\pi$	$1.055 \cdot 10^{-34} \text{ m}^2 \text{ kg s}^{-1}$
Boltzmann's constant	k_B	$1.381 \cdot 10^{-23} \text{ J/K}$ or $8.619 \cdot 10^{-5} \text{ eV/K}$
Radiation constant	$\alpha_{rad} = \pi^2 k_B^4 / 15 \hbar^3 c^3$	$7.565 \cdot 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$
Stefan-Boltzmann constant	$\sigma_{SB} = \pi^2 k_B^4 / 60 \hbar^3 c^2$	$5.670 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Electron rest mass energy	$m_e c^2$	0.511 MeV
Proton rest mass energy	$m_p c^2$	938.3 MeV
Neutron rest mass energy	$m_n c^2$	939.6 MeV
W boson rest mass energy	$m_W c^2$	80.4 GeV
Z boson rest mass energy	$m_Z c^2$	91.2 GeV
Planck energy	$M_{Pl} c^2$	$1.2 \cdot 10^{19} \text{ GeV}$
Thomson cross section	σ_e	$6.652 \cdot 10^{-29} \text{ m}^2$
Neutron half-life (free neutron)	$t_{\frac{1}{2}}$	611 s
Hubble constant	H_0	$100 \cdot h \text{ km s}^{-1} \text{ Mpc}^{-1}$
	h	0.70 ± 0.03
Critical density	ρ_c^0	$1.05 h^2 \cdot 10^{-5} \text{ GeV cm}^{-3}$

Conversion factors

$$\begin{aligned}
 1 \text{ pc} &= 3.261 \text{ light-years} = 3.086 \cdot 10^{16} \text{ m} \\
 1 \text{ AU} &= 1.5 \cdot 10^{11} \text{ m} \\
 1 \text{ year} &= 3.156 \cdot 10^7 \text{ s} \\
 1 \text{ eV} &= 1.602 \cdot 10^{-19} \text{ J} \\
 1 M_{\odot} &= 1.989 \cdot 10^{30} \text{ kg}
 \end{aligned}$$

PARTICLE DATA

(Mass in MeV/c²; Lifetime in seconds; Charge in units of Proton Charge.)

QUARKS (Spin 1/2)

	Flavor	Charge	Mass (speculative)		
			Bare	Effective	
				In baryons	In mesons
First generation	<i>d</i>	-1/3	7.5	363	310
	<i>u</i>	+2/3	4.2		
Second generation	<i>s</i>	-1/3	150	538	483
	<i>c</i>	+2/3	1100	1500	
Third generation	<i>b</i>	-1/3	4200	4700	
	<i>t</i>	+2/3		>23,000	

LEPTONS (Spin 1/2)

	Lepton	Charge	Mass	Lifetime	Principal decays
First generation	<i>e</i>	-1	0.511003	∞	—
	<i>μ</i>	0	~ 0	∞	—
Second generation	<i>μ</i>	-1	105.659	2.197 × 10 ⁻⁶	<i>ν_eν̄_e</i>
	<i>τ</i>	0	~ 0	∞	—
Third generation	<i>τ</i>	-1	1784	3.3 × 10 ⁻¹³	<i>μ_eν̄_e, eν̄_eν̄_e, ρν_e</i>
	<i>ν_e</i>	0	~ 0	∞	—

MEDIATORS (Spin 1)

Mediator	Charge	Mass	Lifetime	Force
gluon	0	0	∞	strong
photon (γ)	0	0	∞	electromagnetic
W [±]	±1	80.4	3 × 10 ⁻²⁵	(charged) weak
Z ⁰	0	91.2	3 × 10 ⁻²⁵	(neutral) weak

BARYONS (Spin 1/2)

Baryon	Quark content	Charge	Mass	Lifetime	Principal decays
<i>N</i> { <i>p</i> <i>n</i> }	<i>uud</i>	+1	938.280	∞	—
	<i>udd</i>	0	939.573	900	<i>pν̄_e</i>
<i>Λ</i>	<i>uds</i>	0	1115.6	2.63 × 10 ⁻¹⁰	<i>pπ⁻, nπ⁰</i>
<i>Σ⁺</i>	<i>uus</i>	+1	1189.4	0.80 × 10 ⁻¹⁰	<i>pπ⁰, nπ⁺</i>
<i>Σ⁰</i>	<i>uds</i>	0	1192.5	6 × 10 ⁻²⁰	<i>Λγ</i>
<i>Σ⁻</i>	<i>dds</i>	-1	1197.3	1.48 × 10 ⁻¹⁰	<i>nπ⁻</i>
<i>Σ⁰</i>	<i>uss</i>	0	1314.9	2.90 × 10 ⁻¹⁰	<i>Λπ⁰</i>
<i>Ξ⁻</i>	<i>dss</i>	-1	1321.2	1.64 × 10 ⁻¹⁰	<i>Λπ⁻</i>
<i>Λ_c⁺</i>	<i>udc</i>	+1	2281	2 × 10 ⁻¹³	not established

BARYONS (Spin 3/2)

Baryon	Quark content	Charge	Mass	Lifetime	Principal decays
<i>Δ</i>	<i>uuu, uud, udd, ddd</i>	+2, +1, 0, -1	1232	0.6 × 10 ⁻²³	<i>Nπ</i>
<i>Σ*</i>	<i>uus, uds, dds</i>	+1, 0, -1	1385	2 × 10 ⁻²³	<i>Δπ, Σπ</i>
<i>Ξ*</i>	<i>uss, dss</i>	0, -1	1533	7 × 10 ⁻²³	<i>Ξπ</i>
<i>Ω⁻</i>	<i>sss</i>	-1	1672	0.82 × 10 ⁻¹⁰	<i>ΔK⁻, Σ⁰π⁻, Ξ⁻π⁰</i>

PSEUDOSCALAR MESONS (Spin 0)

Meson	Quark content	Charge	Mass	Lifetime	Principal decays
<i>π[±]</i>	<i>u\bar{d}, d\bar{u}</i>	+1, -1	139.569	2.60 × 10 ⁻⁸	<i>μν_e</i>
<i>π⁰</i>	<i>(u\bar{u} - d\bar{d})/√2</i>	0	134.964	8.7 × 10 ⁻¹⁷	<i>γγ</i>
<i>K[±]</i>	<i>u\bar{s}, s\bar{u}</i>	+1, -1	493.67	1.24 × 10 ⁻⁸	<i>μν_e, π[±]π⁰, π[±]π⁻π[±]</i>
<i>K⁰, K[±]</i>	<i>d\bar{s}, s\bar{d}</i>	0, 0	497.72	<i>K⁰</i> 0.892 × 10 ⁻¹⁰ <i>K[±]</i> 5.18 × 10 ⁻⁸	<i>π[±]π⁰, π⁰π⁰</i>
<i>η</i>	<i>(u\bar{u} + d\bar{d} - 2s\bar{s})/√6</i>	0	548.8	7 × 10 ⁻¹⁷	<i>ππ, πμν_e, πππ</i>
<i>η'</i>	<i>(u\bar{u} + d\bar{d} + s\bar{s})/√3</i>	0	957.6	3 × 10 ⁻²²	<i>γγ, π⁰π⁰</i>
<i>D[±]</i>	<i>c\bar{d}, d\bar{c}</i>	+1, -1	1869	9 × 10 ⁻¹³	<i>Kππ</i>
<i>D⁰, D[±]</i>	<i>c\bar{u}, u\bar{c}</i>	0, 0	1865	4 × 10 ⁻¹³	<i>Kππ</i>
<i>F[±]</i> (now D [±])	<i>c\bar{s}, s\bar{c}</i>	+1, -1	1071	3 × 10 ⁻¹²	not established
<i>B[±]</i>	<i>u\bar{b}, b\bar{u}</i>	+1, -1	5271	14 × 10 ⁻¹³	<i>D + γ</i>
<i>B⁰, B[±]</i>	<i>d\bar{b}, b\bar{d}</i>	0, 0	5275		
<i>η_c</i>	<i>c\bar{c}</i>	0	2981	6 × 10 ⁻²³	<i>KKπ, ηππ, γππ</i>

VECTOR MESONS (Spin 1)

Meson	Quark content	Charge	Mass	Lifetime	Principal decays
<i>ρ</i>	<i>u\bar{d}, d\bar{u}, (u\bar{u} - d\bar{d})/√2</i>	+1, -1, 0	770	0.4 × 10 ⁻²³	<i>ππ</i>
<i>K*</i>	<i>u\bar{s}, s\bar{u}, d\bar{s}, s\bar{d}</i>	+1, -1, 0, 0	892	1 × 10 ⁻²³	<i>Kπ</i>
<i>ω</i>	<i>(u\bar{u} + d\bar{d})/√2</i>	0	783	7 × 10 ⁻²³	<i>π[±]π⁰, π⁰γ</i>
<i>φ</i>	<i>s\bar{s}</i>	0	1020	20 × 10 ⁻²³	<i>K[±]K⁰, K⁰K⁰</i>
<i>J/ψ</i>	<i>c\bar{c}</i>	0	3097	1 × 10 ⁻²⁰	<i>e[±]e[∓], μ[±]μ[∓], 3π, 7π</i>
<i>D*</i>	<i>cd, d\bar{c}, c\bar{u}, u\bar{c}</i>	+1, -1, 0, 0	2010	>1 × 10 ⁻²²	<i>Dπ, Dγ</i>
<i>Υ</i>	<i>b\bar{b}</i>	0	9460	2 × 10 ⁻²⁰	<i>τ[±]τ[∓], μ[±]μ[∓], e[±]e[∓]</i>