Exam, FK5024, Nuclear & particle physics, astrophysics & cosmology, November 1, 2018

Time 08:00 – 13:00, Room FR4

No tools allowed except calculator (provided at the exam) and the attached formula sheets.

1. (4 p) Consider the following processes. If a process can take place draw a Feynman diagram. If a process cannot take place state a conservation law which is violated by that process.
   (a) $\Omega^- \to K^0 + \pi^-$
   Is not possible (spin and baryon number violated).
   (b) $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$
   Is possible (ordinary weak decay through W^- mediator).
   (c) $\mu^+ + \mu^- \to \tau^+ + \tau^-$
   Is possible (if the energy of the muons is enough to create the heavier τ leptons).
   (d) Decide which of the following statements are true or false. If a statement is false explain why.
   (i) The weak force is dominantly responsible for the decay $\rho^0 \to \pi^+ + \pi^-$.
   Wrong! The large decay width (short lifetime) indicates a strong decay.
   (ii) Violation of the conservation of energy has never been observed in a particle decay.
   Correct! (If a particle decays, it has a finite lifetime τ, and the quantum mechanical uncertainty relation means that the energy may seem violated by $\Delta E \sim \mathcal{O}(\frac{\hbar}{\tau})$. However, this can be interpreted as the fact that the mass of an unstable particle is uncertain, rather than that the energy is not conserved.)

2. (4 p) What are the essential features of the liquid-drop, shell, and collective models of the nucleus? Indicate what properties of the nucleus are well predicted by each model.

3. Explain briefly the following concepts:
   (a) (1 p) Dark energy and the cosmological constant, Λ.
   (b) (1 p) Dark matter.
   (c) (2 p) Big Bang Nucleosynthesis (BBN) and ^{4}He production. Would there be more ^{4}He or less, if the neutron half-life would be smaller?
   Less. All surviving neutrons eventually end up in helium nuclei, and the shorter the neutron lifetime, the less neutrons survive and the ^{4}He production goes down due to the lack of neutrons.

4. (4 p) A μ^- and a μ^+ collide head-on at a laboratory. The μ^+ has an energy 120 GeV. An experiment at the laboratory wishes to study the process $\mu^- + \mu^+ \to Z^0$ and selects a range in possible energy values for the μ^-.
   (a) Estimate the range in energy which the μ^- should possess .
   From the particle table, we read the mass of the Z^0 to be 91.2 GeV/c^2, and the lifetime $\tau = 3 \cdot 10^{-25}$ s. This gives a decay full width of $\Gamma = 2\Delta E = \frac{\hbar}{\tau} \sim 2.2$ GeV, where we roughly have to hit the central energy of the peak in the centre of mass system with $E_{CMS} = m_Z c^2 \pm \Delta E$ to excite the resonance. Thus $s = (p_{\mu^-} + p_{\mu^+})^2 =$

$(91.2 \pm 1.1)^2$. Neglecting the rest mass of muons, this means, now working in the lab frame $(120 + E_{\mu^-})^2 - (120 - E_{\mu^-})^2 = 4 \cdot 120 \cdot E_{\mu^-} = (91.2 \pm 1.1)^2$, which gives $E_{\mu^-} = 17.33 \pm 0.42$ GeV. This is the range to choose if we want to study the Z^0 in this experiment.

(b) The experiment observes $\mu^- + \mu^+ \rightarrow e^+ + e^-$.

The energy of the e^+ is 75 GeV. Choose a single energy value from (a) for the μ^- which belongs to the range of energy estimated in (a) and calculate the angle between the e^+ and e^-.

Let us choose the central value 17.33 GeV for E_{μ^-}. Energy conservation means 120 GeV + 17.33 GeV = 75 GeV + E_{e^-} (where E_{e^-} denotes the energy of the electron in the lab frame). Thus, $E_{e^-} = 62.33$ GeV. Here it is an even better approximation to neglect $e^\pm$ masses, and we use that s is an invariant to compute in the lab frame after the reaction $s = 2 \cdot 75 \cdot 62.33 \cdot (1 - \cos\theta_\pm) = (91.2)^2$, or $1 - \cos\theta_\pm = 0.89$, i.e. $\cos\theta_\pm = 0.11$, or $\theta_\pm = 93^o$.

5. Natural gold ${}^{197}_{79}$Au is radioactive since it is unstable against α-decay with an energy of 3.3 MeV.
(a) (2 p) Is that expected from the Semi-empirical mass formula?
(b) (2 p) Estimate the lifetime of ${}^{197}_{79}$Au to explain why gold does not burn a hole in your pocket.

Useful formulas:

Geiger-Nuttall relation: $\log_{10}\lambda = C - DE_\alpha^{-1/2}$, $C \approx 52$, $D \approx 140$ (MeV)$^{1/2}$
Semi-empirical mass formula (Bethe-Weizsäcker):

$$E_B = a_V A - a_S A^{2/3} - a_A \frac{(A-2Z)^2}{A} - a_C \frac{Z(Z-1)}{A^{1/3}} + \delta(A,Z)$$

with

$$\delta(A,Z) = \left\{ \begin{array}{ll} +\delta_0 & \mathrm{N, Z\ even, A\ even} \\ 0 & \mathrm{A\ odd} \\ -\delta_0 & \mathrm{N, Z\ odd, A\ even} \end{array} \right\}, \delta_0 = \frac{a_P}{A^{1/2}}$$

Volume term: $a_V = 15.85$ MeV
Surface term: $a_S = 18.34$ MeV
Asymmetry term: $a_A = 23.21$ MeV
Coulomb term: $a_C = 0.714$ MeV
For pairing term: $a_P = 12.00$ MeV

(a) Using the given SEMF one finds $Q = -E_B(197,79) + E_B(193,77) + E_B(4,2) = (-1566.2 + 1540.6 + 22.3)$ MeV $= -3.3$ MeV, which is negative, so the SEMF does not work in this case (one has to add the extra binding energy of the alpha particle of around 6 MeV - not given in the problem text - which gives a positive value around 3 MeV).

(b) The Geiger-Nuttall relation gives, for $E = 3.3$ MeV, $\log_{10}\lambda = 52 - \frac{140}{\sqrt{3.3}} \approx -25.1$ s^{-1}. With the decay law $N(t) = N_0 e^{-\lambda t}$ we see that the natural decay time scale is $1/\lambda \sim 10^{25.1}$ s, which is far longer than the age of the universe $t_0 \sim 4 \cdot 10^{17}$ s. Natural gold is regarded as stable, due to the Coulomb barrier for α-decay.

6. (a) (2 p) An alternative to the cosmological constant may be an unusual "fluid" U, the density of which changes with the scale factor like $\rho_U(t) \sim 1/a(t)$. What is the equation of state parameter w_U for this fluid?

From the fluid equation in the formula sheet one gets

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a}$$

which can be integrated with respect to time to give $\ln(\rho) = -3(1+w)\ln(a)+\text{const.}$ Exponentiation then gives $\rho \sim a^{-3(1+w)}$. Here it was given that $\rho \sim a^{-1}$, meaning $-3(1+w) = -1$, or $w = -2/3$

(b) (2 p) Assume that the matter and U content now corresponds to $\Omega_M = 0.3$ and $\Omega_U = 0.7$ (i.e., a flat universe with no cosmological constant and where radiation can be neglected). Compute the value of the "deceleration parameter" for this hypothetical universe.

From the formula sheet, we find $q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)}\frac{1}{H_0^2}$, and $\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\sum_i\left(\rho_i + 3\frac{p_i}{c^2}\right) = -\frac{4\pi G_N}{3}\sum_i \rho_i(1+3w_i)$. With $\rho_c^0 = (3H_0^2)/(8\pi G_N)$ (also from the formula sheet) we can find (for $z = 0$, and using $w_M = 0$, $\Omega_U = -2/3$,) $q_0 = \frac{1}{2}\left[\Omega_M - \Omega_U\right] = -0.2$ (The deceleration parameter is thus negative, meaning an accelerating universe, but slower acceleration compared to a cosmological constant).