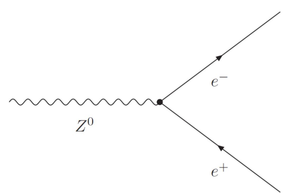
# Exam, FK5024, Nuclear & particle physics, astrophysics & cosmology, October 26, 2017

**08:00 – 13:00, Room FR4 (Oskar Klein Auditorium)**

*No tools allowed except calculator (provided at the exam) and the attached formula sheet.*

1. (4 p) Consider the following decays/reactions (the particles are not bound or virtual). Discuss which of these are possible to observe and draw a Feynman diagram in that case. If a process is impossible state a conservation law forbidding it.
   (a) $Z^0 \to e^+ + e^-$

   Allowed. It is one of the main decays of $Z^0$.

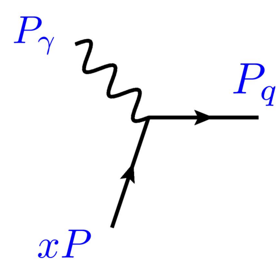
   
   

   (b) $p \to n + e^+ + \nu_e$
   Not allowed because of energy conservation ($m_p c^2 < m_n c^2$).
   (c) $\mu^- \to e^- + \mu^+ + e^-$
   Not allowed (both energy and lepton number conservation).
   (d) $\tau^+ \to e^+ + \nu_\tau + \nu_e$
   Not allowed, lepton number conservation ($\nu_\tau$ should have been $\overline{\nu}_\tau$).

2. (4 p) In a scattering process, an electron interacts with a quark in the proton via the exchange of a virtual photon with four-momentum of the photon $P_\gamma$ (one can show that $P_\gamma^2 < 0$ for scattering). If the proton's four-momentum is $P$, show that the fraction $x$ of the proton's momentum carried by the struck quark is

   $$x = \frac{-P_\gamma^2}{2P \bullet P_\gamma}.$$

   (The symbol $\bullet$ denotes the 4-scalar product.) You can assume that the proton, and thus the quarks, are travelling at a relativistic speed and that particle rest masses can be neglected.
   Look at the photon–quark interaction.




Four-momentum conservation gives $P_\gamma + xP = P_q$, with $xP$ being the 4-momentum of the quark before and $P_q$ the 4-momentum of the quark after the collision. Squaring gives $P_\gamma^2 + 2xP \bullet P_\gamma = 0$ since masses can be neglected so $P^2$=$P_q^2$=0. Thus

$$x = \frac{-P_\gamma^2}{2P \bullet P_\gamma}.$$

3. (4 p) The maximum positron kinetic energy in the spectrum of positrons emitted in the nuclear decay $^{11}\mathrm{C} \to ^{11}\mathrm{B}$ is 0.96 MeV. Use this information and the known mass of $^{11}$B, 10.2551 GeV/$c^2$, to compute the mass of $^{11}$C.
In the decay, a neutrino is also emitted, and the positron and neutrino share the energy. Thus, when the positron kinetic energy is maximal, the neutrino gets minimal energy which is very close to zero, as neutrino masses are extrememly tiny. Thus we can forget about the neutrino in the calculation. The positron kinetic energy is its total energy minus its rest mass energy, i.e., $E_{kin} = E_e^{tot} - m_e c^2$ or $E_e^{tot}$ = 0.96 MeV+0.511 MeV = 1.47 MeV. The absolute value $k$ of the momentum of the positron and the boron nucleus are equal and the direction is opposite (as the $C$ nucleus decays at rest), and energy conservation gives

$$M_C c^2 = \sqrt{M_B^2 c^4 + (kc)^2} + E_e^{tot}.$$

As the value of $kc$ is of the order of an MeV, we can neglect that contribution in the square root, and we simply get $M_C c^2 = M_B c^2 + E_e^{tot}$ = 10.2551 GeV + 0.00147 GeV = 10.257 GeV.

4. Consider a parent nucleus with Z+2 protons, undergoing $\alpha$-decay into a daughter nucleus with $Z$ protons. The charge of the $\alpha$-particle in units of $e$ is $z = 2$.
(a) (1 p) Write down the expression for the Coulomb potential $V(r)$ of the $\alpha$-particle at a distance $r$ from the daughter nucleus.
From the formula sheet ($Z_1 = 2$, $Z_2 = 90$),

$$V(r) = \frac{2Z\alpha\hbar c}{r} = \left(\frac{2Z}{r}\right) \cdot \alpha\hbar c = \frac{2.88Z \text{ MeV} \cdot \text{fm}}{r}$$

(b) (1 p) Given the binding energy $B = 34\,\text{MeV}$ and $Z = 90$, find the value of the radius $a$ where the $\alpha$-particle is classically confined using the formula $B = V(a)$. Given the value $Q = 6\,\text{MeV}$ for the reaction, find the value of the radius $b$ for which the $\alpha$-particle has tunneled away of the Coulomb potential, using the formula $Q = V(b)$.

$B = V(a) \Rightarrow 34$ MeV $= 2.88 \cdot 90$ MeV·fm$/a \Rightarrow a = 259/34$ fm $= 7.62$ fm.

$Q = V(b) \Rightarrow 6$ MeV $= 259$ MeV·fm$/b \Rightarrow b = 259/6$ fm $= 43.2$ fm.

(c) (2 p) Compute the Gamow factor

$$G \approx \sqrt{\frac{2mc^2}{Q}}\frac{zZ}{137}\left(\frac{\pi}{2} - 2\sqrt{x}\right),$$

where $x = a/b = Q/B$, and $mc^2 = 3.73\,\text{GeV}$ is the rest mass of the $\alpha$-particle. Estimate the (small) probability to penetrate the barrier, using the formula $P = \exp(-2G)$.

$x = Q/B = 6/34 = 0.176$, $z = 2$, $Z = 90$, $Q = 6$ MeV, $mc^2 = 3730$ MeV gives, using the calculator, $G = 33.8$. Thus,

$$P = e^{-2G} = 4.1 \cdot 10^{-30}$$

(The probability per unit time is still significant, since the alpha particle moves fast and therefore hits the barrier many times before escaping.)

5. Explain briefly the following concepts:

(a) (1 p) Radiation-matter equality.

The epoch in the early uinverse when the energy density of radiation equaled that of the matter. (At $z \sim 10^4$.)

(b) (1 p) Dark matter.

The presently unknown component of the matter density that does not emit or absorb light. (So-called WIMPs and axions are presently two of the main candidates for dark matter.)

(c) (2 p) The geometry term proportional to $k$ and the $\Lambda$ term in the Friedmann equation.

$k$ gives information on the geometry: $k = 0$ means the universe is geometrically flat on large scales, $k > 0$ that it is closed, $k < 0$ means an open universe. $\Lambda$ is a measure of the cosmological constant (vacuum energy, or "dark energy").

6. (4 p) (a) (1 p) For an equation of state $p/c^2 = w \cdot \rho$, how does $\rho$ depend on the scale factor $a$?

From the fluid equation in the formula sheet one gets

$$\frac{\dot\rho}{\rho} = -3(1+w)\frac{\dot a}{a}$$

which can be integrated with respect to time to give
$\ln(\rho) = -3(1+w)\ln(a)$+const. Exponentiation then gives $\rho \sim a^{-3(1+w)}$.

(b) (2 p) The early Universe could in principle be dominated by cosmic strings of length $l = a(t) \cdot l^0$ where $a(t)$ is the scale factor and $l^0$ is a constant (i.e., the strings get longer by the scale factor). The energy density in a string is $\lambda \cdot l$ with the string tension $\lambda$ being a constant. The total energy density of $i = 1, 2, 3, \ldots$ strings of lengths $l_i = a(t) l_i^0$ in a physical volume $V$ is thus

$$\rho_s = \sum_i \frac{\lambda l_i}{V}.$$

What is the equation of state for these cosmic strings?
$\lambda l_i \sim a$ and $V \sim a^3$ gives $\rho_s \sim a^{-2}$. Comparing this to the result in (a) gives $3(1+w) = 2$, or $w = -1/3$.
(c) (1 p) How does the scale factor depend on time for cosmic strings?
From the acceleration equation in the formula sheet, and the result in (b), $p/c^2 = w \cdot \rho = -\rho/3$ one finds

$$\frac{\ddot{a}}{a} = 0,$$

i.e. $a(t) \sim t$.

**Good Luck!**